Journal of ISOSS
2017 Vol. 3(1), 15-34

EMPLOYING CCA IN DETERMINING THE SIGNIFICANT CAUSAL RELATIONSHIPS AMONG STATISTICAL COURSES IN BENGHAZI UNIVERSITY

Osama H. Othman[1], **Abdelghffar. F Abdelghffar**[2], **Rami Salah Gebril**[2] and **Ashraf A. Ahmed Younis**[2]
[1] Department of Statistics, Faculty of Arts and Science, University of Ajdabiya, Ajdabiya, Libya.
[2] Department of Statistics, Faculty of Science, University of Benghazi, Benghazi, Libya.

ABSTRACT

In the context of seeking and detecting the causal relationships between different variables and phenomena, Canonical Correlation Analysis (CCA) is one of the extensions of Regression Analysis that serves this purpose. This multivariate technique was employed to explore causality among the marks of students achieved in the Statistical courses. Our work consists of building a series of models using the courses studied in first several semesters considering them as independent variables and the subsequent courses as dependent variables, and exploring the existence of significant relationships between these two categories of courses. The number of variables and complexity of the models will increase gradually as the order of semesters rises, and that is due to the gradual addition of number of courses in models, until all courses are submitted in the final model. In this study, the main data matrix will have the order 41×26, and will consist of the Grade Point Average (GPA) of students in 26 courses (variables) in the Faculty of Science at the University of Benghazi. It was found that there is a significant dependence between statistical courses by their subsequent order in several models using Canonical Correlation Analysis.

KEYWORDS

Canonical Correlation Analysis, Causal Relationship, Redundancy Analysis.

INTRODUCTION

In this work, we attempt to explore the existence of a statistical significance dependence among the courses of the Department of Statistics at the University of Benghazi considering their subsequent order throughout the whole period of study organized by the department. That task can be achieved by using the technique of Canonical Correlation Analysis (CCA), which can interpret and clarify mutual hidden relationships among the variables included in a model. The number of compulsory and elective statistical courses in the Department of Statistics are 26 courses, and are showed in Table 1 with their corresponding codes.

Table 1
Names and Codes of the Variables Included in the Study

S.No.	Codes of the Variables	Courses Names
1	X31s5	Advanced Distribution Theory
2	X32s4	Distribution Theory
3	X33s5	Estimation Theory
4	X34s6	Probability Theory
5	X35s8	Graduation Project
6	X36s3	Elements of Probability 2
7	X37s2	Elements of Probability 1
8	X38s6	Stochastic Process
9	X39s7	Nonparametric Methods
10	X40s8	Applied Linear Models
11	X41s5	Experimental Design 2
12	X42s4	Experimental Design 1
13	X43s8	Time Series Analysis
14	X44s7	Data Analysis
15	X45s5	Regression Analysis
16	X46s6	Operation Research
17	X47s4	Demography
18	X48s6	Multivariate Analysis
19	X49s1	General Statistics
20	X50s4	Applied Statistics
21	X51s5	Sampling Techniques 2
22	X52s4	Sampling Techniques 1
23	X54s6	Test of Hypothesis
24	X55s6	Bayesian Statistics
25	X56s7	Biostatistics
26	X57s3	Statistical Methods

METHODOLOGY

Suppose that R_{XX} and R_{YY} represent the correlation matrices for the independent variables group (X) and dependent variables group (Y) respectively, and represents the correlation matrix between the two groups (X) and (Y). Let $X^* = a'X$ and $Y^* = b'Y$ be the linear combinations of the independent variables (X) and the dependent variables (Y) respectively. Then the values of the constant vectors (a, b) are determined so that the correlation coefficient between each pair of the corresponding linear combinations (X^*, Y^*) becomes a maximized value; that is:

$$r^2 = max_{a,b} r^2 X^* Y^*$$

The vectors (a, b) represent Eigen vectors corresponding to the following characteristic equations:

$$(R_{XX}^{-1} R_{YX}' R_{YY}^{-1} R_{YX} - r^2 I)a = 0 \text{ , } (R_{YY}^{-1} R_{XY}' R_{XX}^{-1} R_{XY} - r^2 I)b = 0$$

For achieving trusted results with CCA, the significance of correlation between the two groups of variables of the study needs to be checked, and that can be done by testing the next statistical hypothesis:

$$H_0: \sum YX = 0$$

This hypothesis means that the group of canonical independent variables $X^* = a'X$ is independent of the group of canonical dependent variables $Y^* = b'Y$and in this case it is not suitable to perform CCA to analyze the underlying data. To test this hypothesis the following formula of Chi-Square statistic is used:

$$\chi^2 = -\left[(n-1) - \frac{1}{2}(p+q+1)\right] ln\, \Lambda = -\left[n - \frac{1}{2}(p+q+3)\right] ln\, \Lambda$$

where Λ represents the percentage of variation or correlation between variables to the percentage within variables, (or can be also defined as the geometric sum of 1 minus the squared canonical correlation). The squared canonical correlation is an estimate of the common variance between two canonical variates. Thus, 1 minus this value is an estimate of the unexplained variance. Again, Lambda is used as a test of significance for the squared canonical correlation and equals to:

$$\Lambda = \prod_{i=1}^{K}\left(1 - \lambda_{(i)}\right) = \prod_{i=1}^{K}(1 - r_{(i)}^2)\ , i = 2, \dots, K$$

where $\boldsymbol{\lambda_i}$ or $\boldsymbol{r_i}^2$ represent the i^{th} Eigen value of the matrix $S_{YY}^{-1}S_{YX}S_{XX}^{-1}S_{XY}$ or $S_{XX}^{-1}S_{XY}S_{YY}^{-1}S_{YX}$ and K = min (p,q), where p and q are the number of independent and dependent variables respectively. It is worth noting that the previous statistic belongs to Chi-Square distribution with $v = p \times q$ degrees of freedom in cases that involve large samples.

The next step is checking that all canonical correlations are significant to describe the linear relationship between the two groups of variables. That can be done via significance testing of each pair of the canonical variables. To test the significance of $K' < K = \min(p, q)$ canonical pairs, we need to carry out the procedure of the same previous test; that is to test the next statistical hypothesis each time:

$$H_0: \rho_1 \neq 0, \rho_2 \neq 0, \dots, \rho_{K'-1} \neq 0, \rho_{K'} = 0, \dots, \rho_K = 0\ , K' < K = \min(p, q)$$

which investigates that the first (k-1) of canonical variables pairs are statistically significant and the rest of them are not. Again, the Chi-Square statistic can be re-written as follows:

$$\chi^2 = -\left[(n-1) - \tfrac{1}{2}(p+q+1)\right] ln\, \Lambda^* = -[\left(n - \tfrac{1}{2}(p+q+3)\right] ln\, \Lambda$$

where $\Lambda^* = \prod_{i=K'+1}^{K}\left(1 - \lambda_{(i)}\right) = \prod_{i=K'+1}^{K}(1 - r_{(i)}^2)$ and $K' < K = \min(p, q)$

with $v' = (q - K')(p - K')$ degrees of freedom.

If the null hypothesis was rejected, then the K' canonical pairs are all significant in describing the relationships between groups of variables. Worth noting is that we put $\left(K' = 1\right)$, then we test the significant of the second canonical variable pairs; in this case the test statistic will be:

$\left(K^{'}=1\right),\ \chi^2 = -\left[(n-1) - \frac{1}{2}(p+q+1)\right] ln\ \Lambda_1^*$

with $\upsilon = (p-1)(q-1)$ degrees of freedom. The effect of the first canonical variable pair will be removed from Λ to became as follows:

$$\Lambda_1^* = \prod_{i=2}^{K}\left(1-\lambda_{(i)}\right) = \prod_{i=2}^{K}(1-r_{(i)}^2)$$

Since the main objective of using the CCA is studying the relationship between two groups of variables, then the amount of association can be known by Canonical Gross weight, which represents the amount of simple correlation between each variable in the dependent group (Y) directly with the independent canonical variable $a'X$, and also between each variable in the independent group (X) directly with $b'Y$. That measure can be obtained by multiplying canonical loadings by canonical correlation coefficient, hence the correlation coefficient between the dependent variable $\boldsymbol{Y_j}$ and $\boldsymbol{i^{th}}$ variable in the independent group will be expressed as $r_{X^*}Y_j^{(i)}$ and can be estimated as follows:

$$r_{xy_j}(i) = R_{yy_j}(i)\ r_i = R_{yy_j}(i)\sqrt{\lambda_i}\ , i = 2, \dots, K$$

where $\boldsymbol{r_i}$ represents the $\boldsymbol{i^{th}}$ canonical correlation coefficient which equals to the square root of the $\boldsymbol{i^{th}}$ Eigen value $\boldsymbol{\lambda_i}$ .Likewise the correlation coefficient for the variable $\boldsymbol{X_i}$ with the $\boldsymbol{i^{th}}$ canonical variable in the dependent group (Y) is:

$$r_{yx_i}(i) = R_{xx_i}(i)\ r_i = R_{xx_i}(i)\sqrt{\lambda_i}$$

where $R_{X^*X}(i) = R_{XX}\hat{a}_i$ represents the canonical loadings coefficients for group (X), and $R_{Y^*Y}(i) = R_{YY}\hat{b}_i$ represents canonical loadings for group (Y) which corresponds to the $\boldsymbol{i^{th}}$ canonical variables pair. The percentage of explained variance of the dependent group by the independent group and vice versa. Where the percentage of variance in the dependent group explained by the $\boldsymbol{i^{th}}$ dependent canonical variable is:

$$R_{(i)y}^2 = \frac{1}{P}R_{yy}^{'}(i)R_{yy}(i) = \frac{1}{P}\sum_{j=1}^{p}[R_{yy_j}(i)]^2$$

where $R_{Y^*Y_j}(i)$ represents canonical loadings coefficient for the $\boldsymbol{j^{th}}$ dependent variable in $\boldsymbol{i^{th}}$ canonical variable. Likewise, the percentage of variance in the independent group explained by the $\boldsymbol{i^{th}}$ independent canonical variable is:

$$R_{(i)x}^2 = \frac{1}{q}R'_{xx}(i)R_{xx}(i) = \frac{1}{q}\sum_{i=1}^{q}[R_{xx_i}(i)]^2$$

where $R_{X^*X_i}(i)$ represents canonical loadings coefficient for the $\boldsymbol{i^{th}}$ independent variable in the $\boldsymbol{i^{th}}$ canonical variable .The two previous formulas represent what so called Redundancy Analysis[1][3].

DATA AND RESULTS

The data used in this study is the Grade Point Average (GPA) of students at the Department of Statistics in the Faculty of Science at the University of Benghazi in Libya.

DATA DESCRIPTION

Courses								
Semester		**X31s5**	**X32s4**	**X33s5**	**X34s6**	**X35s8**	-----	**X57s3**
1990-1991	**Fall**	1.27	2.05	1.23	1.79	1.68	-----	1.07
1990-1991	**Spring**	1.42	1.85	1.36	2.45	0.67	-----	1.82
1991-1992	**Fall**	2.19	1.72	2.85	2.21	2.43	-----	3.02
1991-1992	**Spring**	1.97	1.75	2.32	2.14	2.07	-----	1.63
-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-
-	-	-	-	-	--	-	-	-
2009-2010	**Spring**	1.65	1.64	2.49	0.95	1.62	-----	2.67

Figure 1: A Brief Description of the Grade Point Average Data Matrix under the Study

Figure 1 shows the construction of the data matrix where its columns contain the courses (variables) and the rows contain the semesters (cases). Our empirical analysis was performed on 41 semesters, (denoted by S), from the Fall semester of the academic year (1990/1991) to the Spring semester of the academic year (2009/2010). As for the number of variables, only 251 out of 616 courses were chosen as they were the most studied courses in the whole duration of the study. The data in the cells represent the average of marks for all students in the semesters represented horizontally for the courses (variables) represented vertically. Every variable (course) in the data will be indexed as follows:

- The first index refers to the serial number of the variable and the count is from 1 to 251.
- The second index refers to the semester in which that course must be taken or studied.

The application begins with using some tools and measures of descriptive statistics such as the Mean, Standard Deviation, and the Boxplot to explore their distribution and to highlight the outliers. Table 2 shows the 26 variables of the study with their mean and standard deviation.

Table 2
The Mean and Standard Deviation for the Variables of the Study

Variables	Mean	Standard Deviation
X31s5	1.639024	0.339999
X32s4	1.391463	0.403371
X33s5	1.630976	0.229312
X34s6	1.999756	0.447926
X35s8	1.941707	0.612605
X36s3	1.521463	0.532487
X37s2	1.581220	0.484867
X38s6	2.033171	0.387101
X39s7	1.946341	0.359574
X40s8	1.457073	0.513791
X41s5	1.675610	0.484211
X42s4	1.674390	0.377763
X43s8	2.182195	0.401532
X44s7	2.655610	0.425559
X45s5	1.559268	0.454001
X46s6	1.851220	0.448760
X47s4	2.079512	0.570710
X48s6	1.792439	0.323719
X49s1	1.269512	0.488983
X50s4	1.913171	0.653110
X51s5	1.819756	0.619268
X52s4	1.812683	0.597917
X54s6	1.561463	0.320933
X55s6	2.060488	0.442707
X56s7	1.536585	0.301020

Figure 2 demonstrates several features such as the variation between and within the grades of students which appears to change slightly between some courses (as in those located on the right and left of the plot), and to change more sharply in the middle courses. The symmetry property can be detected via that plot, and it is clear that the majority of variables are approximately symmetric.

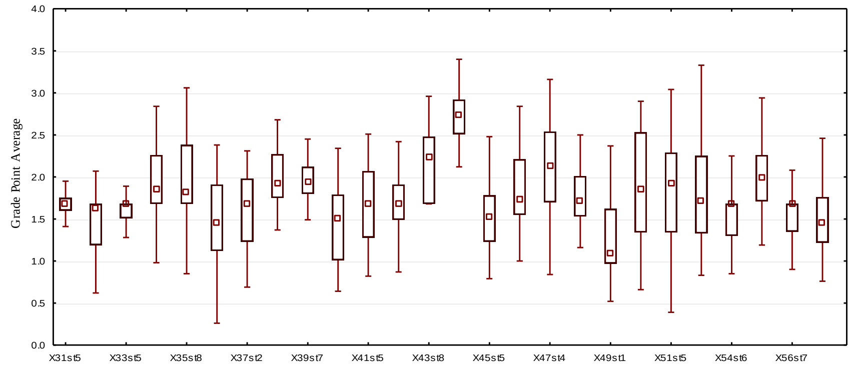

Figure 2: Box Plot Describing the Distribution of Variation between Variables

DATA MODELING

The considerations in this section are for variables and models' preparation; the next table, (Table 3), shows the division of our 26 variables in the department into four models. As can be seen throughout the first model to the fourth model, the number of independent variables increases because each dependent variable in the first three models will be independent in the next model until the fourth.

Table 3
The Included Models and Variable in the Study

First Model		Second Model		Third Model		Fourth Model	
Independent	Dependent	Independent	Dependent	Independent	Dependent	Independent	Dependent
X36s3	X32s4	X32s4	X33s5	X32s4	X34s6	X32s4	X35s8
X37s2	X42s4	X36s3	X31s5	X36s3	X38s6	X36s3	X39s7
X49s1	X47s4	X37s2	X41s5	X37s2	X46s6	X37s2	X40s8
X57s3	X52s4	X42s4	X45s5	X42s4	X48s6	X42s4	X43s8
	X50s4	X49s1	X51s5	X49s1	X54s6	X49s1	X44s7
		X47s4		X47s4	X55s6	X47s4	X56s7
		X50s4		X50s4		X50s4	
		X52s4		X52s4		X52s4	
		X57s3		X57s3		X57st3	
				X31s5		X31s5	
				X33s5		X33s5	
				X41s5		X41s5	
				X51s5		X51s5	
				X45s5		X45s5	
						X34s6	
						X38s6	
						X46s6	
						X48s6	
						X54s6	
						X55s6	

CANONICAL CORRELATION ANALYSIS APPLICATION

Starting with the preliminary tests and calculations for CCA, the results of Canonical R, Canonical R square, the Chi-Square test and the Lambda statistics for the four models included in the analysis are shown in Table 4. From this table, it can be seen that the Canonical R and Canonical R square values for all models are sufficient to explain a large amount of the variation of the causal relationships among courses. In addition, the P-values of the first removed root for the four models are significant and that allows us to move forward. The Lambda statistic shows a decreasing pattern for the percentage of variation between courses to the percentage within courses. It can be noticed also that as the number of involved courses through the models increase, the Lambda value decreases and that means the estimation of unexplained variance is decreasing as we involve more courses in the models. At this stage, it can be said that the four models were significant for the first removed root.

Table 4
The Chi-Square test Summary of First Roots Removed for all Models

	Canonical R	Canonical R^2	Chi-square	d.f.	p-value	Lambda
First Model	0.693651	0.481151	48.66217	20	0.000346	0.248989
Second Model	0.794392	0.631058	71.80130	45	0.006791	0.109781
Third Model	0.891586	0.794925	124.1828	84	0.002937	0.014852
Fourth Model	0.899224	0.808603	157.8370	120	0.011872	0.002590

The next procedure is about Redundancy Analysis and variance extracted percentage which can be considered as a descriptive way to figure out the amount of association or correlation between the group of independent and dependent variables relative to the amount of variance in them. The variance extracted can be interpreted as the average amount of variation from the variables in each set by all canonical roots, as it is described in table 5. Note that one of the values will always be 100% because the STATISTICA software extracts as many roots as the minimum number of variables in either group. Thus, for one group of variables, there are as many independent sums (canonical variates) as there are variables. The total redundancy can be interpreted on the basis of all canonical roots, whereas each group of variables gives on the average the percentage of variation that can be accounted for in the other group of variables as shown in table 5.

Table 5
Redundancy analysis and variance explained for the four models.

	First Model		Second Model		Third Model		Fourth Model	
	Dependent	Independent	Dependent	Independent	Dependent	Independent	Dependent	Independent
Variance Extracted	83.24%	100%	100%	62.51%	100%	53.46%	100%	37.99%
Total Redundancy	27.50%	30.19%	34.20%	23.50%	49.69%	26.16%	62.94%	25.29

The canonical weights and their contribution to the first removed root of the First Model are given in Table 6. Obviously, the variable $\mathbf{X_{37s2}}$, $\mathbf{X_{49s1}}$, $\mathbf{X_{32s4}}$ and $\mathbf{X_{50s4}}$ have the largest contribution to the first removed root. Other tables which show the canonical weights for the rest of models can be found in Appendix A.

Table 6
The Canonical Weights of the First Removed Root for the First Model

Variables	Canonical Weights for Independent Variables (X)	Canonical Weights for Independent Variables (Y)
X36s3	-0.245631	
X37s2	0.616580	
X49s1	0.770347	
X57s3	-0.270735	
X32s4		-0.775486
X42s4		0.083776
X47s4		-0.224177
X50s4		-0.785108
X52s4		0.054757

The canonical factor loadings are exhibited in Table 7 for first model by giving the correlations between the variables and the first canonical root, the amount of explained variance for the first canonical root is shown as an indication for the amount of variation that the first canonical root could explain for the two groups of variables. Redundancy values were calculated to figure out the amount of variation for the first removed root with the two groups of variables. From Table 7 it can be noted that the independent variables explains 12.2% of the variation in the dependent variables for the first removed root and the dependent variables explained 9.7% of the variation in the independent variables for the first removed root. The rest of the tables that contain the results of canonical factor loadings are displayed in the appendix A.

Table 7
The Canonical Factor Loadings of the Two Groups (X) and (Y), and the Percentage of Explained Variance for each Group with the Corresponding Canonical Variable for the First Model

Variables	Canonical Factor Loadings for Independent Variables (X)	Canonical Factor Loadings for Dependent Variables (Y)
X36s3	0.065060	
X37s2	0.631806	
X49s1	0.773592	
X57s3	-0.112612	
X32s4		-0.411591
X42s4		0.183264
X47s4		-0.580878
X50s4		-0.678899
X52s4		0.040819
Variance Explained	25.4%	20.1%
Redundancy	12.2%	9.7%

Table 8 shows the last step of CCA by calculating the mixed factor loadings that represent the direct relationship between the independent and dependent variables. The independent variables which correlate highly in a negative fashion are $\mathbf{X_{37s2}}$ and $\mathbf{X_{49s1}}$ with the dependent variables $\mathbf{X_{32s4}}$ and $\mathbf{X_{50s4}}$. The meaning of these associations is that the behavior of students' grades on average is expressed in a negative way, reflecting that the average results of the student in the courses of the 1st, 2nd and 3rd semesters are high and then became lower in the courses of the 4th semester.

Table 8
Mixed Factor Loadings of Variable groups of First Removed Root for the First Model

Variables	Mixed Factor Loadings independent Variables Group (X)	Mixed Factor Loadings Dependent Variables Group (Y)
X36s3	0.045129	
X37s2	0.438253	
X49s1	0.536603	
X57s3	-0.07811	
X32s4		-0.53792
X42s4		0.058111
X47s4		-0.1555
X50s4		-0.54459
X52s4		0.037982

The mixed factor loadings for the second model, (represented in Table 9), show that the independent variables $\mathbf{X_{32s4}}$, $\mathbf{X_{36s3}}$, $\mathbf{X_{42s4}}$, $\mathbf{X_{52s4}}$ are associated positively with the dependent variables $\mathbf{X_{51s5}}$, $\mathbf{X_{41s5}}$ and $\mathbf{X_{31s5}}$ which means that the students' grades are increasing in the two groups of variables. The previous three dependent variables correlate negatively with the independent variable $\mathbf{X_{50s4}}$ indicating that the performance of the students was not high in the course $\mathbf{X_{50s4}}$ and this performance was improved in the subsequent courses of the 5th semester.

Table 9
The Mixed Factor Loadings of the two Variable Groups of first Removed Root for the Second Model.

Variables	Mixed Factor Loadings independent Variables Group (X)	Mixed Factor Loadings Dependent Variables Group (Y)
X32s4	0.411595	
X36s3	0.489012	
X37s2	0.143721	
X42s4	0.655831	
X47s4	-0.16867	
X49s1	0.020259	
X50s4	-0.38806	
X52s4	0.454549	
X57s3	0.061229	
X31s5		0.203187
X33s5		0.136666
X41s5		0.470354
X45s5		0.037047
X51st5		0.673397

In Table 10, it can be found that most of the mixed factor loading of the independent variables group is negative and most of the dependent variables group is positive. This result assumes that the performance of the students on average was low in the courses of the semesters 1 to 5, and then improves in later courses of the 6th semester. These facts can be seen obviously in the independent variables $\mathbf{X_{52s4}}$, $\mathbf{X_{36s3}}$, $\mathbf{X_{57s3}}$ and $\mathbf{X_{32s4}}$ which associated negatively with the dependent variables $\mathbf{X_{48s6}}$, $\mathbf{X_{34s6}}$ and $\mathbf{X_{46s6}}$. The previous independent variables associated in a positive way with the dependent variable $\mathbf{X_{54s6}}$ and this indicates that the performance of the students is decreasing.

Table 10
The Mixed Factor Loadings of the Two Variable Groups of first Removed Root for the Third Model.

Variables	Mixed Factor Loadings Independent Variables Group (X)	Mixed Factor Loadings dependent Variables Group (Y)
X31s5	0.189084	
X32s4	-0.35148	
X33s5	0.186597	
X36s3	-0.52072	
X37s2	-0.372	
X41s5	-0.0396	
X42s4	0.009703	
X45s5	-0.16238	
X47s4	0.467014	
X49s1	-0.22977	
X50s4	0.359339	
X51s5	-0.27533	
X52s4	-0.69762	
X57s3	-0.46457	
X34s6		0.606784
X38s6		0.371366
X46s6		0.58395
X48s6		0.652512
X54s6		-0.41424
X55s6		0.177916

Many interpretations and results can be found in the three previous tables which represented the outcomes of mixed factor loadings, and we conclude this section by representing Table 11 which is the last table that shows the mixed factor loadings of the fourth model and contains all the variables of the study.

Table 11
Mixed Factor Loadings of the two Variable Groups of First Removed Root for the Fourth Model

Variables	Mixed factor Loadings Independent Variables Group (X)	Mixed Factor Loadings Dependent Variables Group (Y)
X31s5	-0.03912	
X32s4	-0.41039	
X33s5	0.052849	
X34s6	0.04039	
X36s3	-0.45278	
X37s2	-0.25725	
X38s6	0.160655	
X41s5	-0.59175	
X42s4	-0.37605	
X45s5	0.183305	
X46s6	0.206912	
X47s4	0.280066	
X48s6	-0.06159	
X49s1	-0.20283	
X50s4	0.582792	
X51s5	-0.24271	
X52s4	-0.19799	
X54s6	-0.31606	
X55s6	-0.13597	
X57s3	-0.29468	
X35s8		0.240852
X39s7		-0.22796
X40s8		-0.53006
X43s8		0.51024
X44s7		0.077149
X56s7		-0.6598

From Table 11, a very interesting mix of relationships can be observed between independent and dependent variables via their mixed factor loadings, and our interpretation will focus on the variables that hold high loadings. Few variables have positive increasing relationships in the two groups. For instance, the independent variable $\mathbf{X_{50s4}}$ associated in a highly positive fashion with the dependent variable $\mathbf{X_{43s8}}$ and at the same time, associated in a negatively with the dependent variables $\mathbf{X_{56s7}}$, $\mathbf{X_{40s8}}$ and $\mathbf{X_{39s7}}$. Some other variables correlate negatively with each other in the two groups such as the independent variables $\mathbf{X_{41s5}}$, $\mathbf{X_{36s3}}$, $\mathbf{X_{32s4}}$, $\mathbf{X_{42s4}}$, $\mathbf{X_{54s6}}$ and $\mathbf{X_{57s3}}$ which correlate negatively with the dependent variables $\mathbf{X_{56s7}}$, $\mathbf{X_{40s8}}$ and $\mathbf{X_{39s7}}$; that is they have the same direction or students' grades behavior in a decreasing way. The rest of variables in the two groups correlate with each other by different signs and that means having different direction. In other words, the performance of the students increases on the average in some courses (independent variables) and decreases in some courses in the succeeding semesters (dependent variables). Another situation can be seen with the variables in the two groups that have the same sign for their loadings and that means the performance of the students on average increases if the sign is positive or they decrease together if the sign is minus.

The Redundancy Index is computed and displayed in Table 12 for the purpose of describing the total percentage of variation that was explained for the four models in the two groups of variables. After taking the sums for the redundancy index for the four models for both groups, it can be concluded that the group of independent variables can explain 67.21% of variation in the group of dependent variables and the group of dependent variables can explain 48.06% of variation in the group of independent variables.

Table 12
Redundancy Index of the two groups of variables for all models

Models	**Variables group**	λ_1	$R^2 Y$	$R^2 X$	**Redundancy Index**	
					$\lambda_1 \times R^2 Y$	$\lambda_1 \times R^2 X$
First Model	Independent (X)	0.481151		0.254		0.122212
	Dependent(Y)	0.481151	0.201		0.096711	
Second Model	Independent (X)	0.631058		0.2199		0.13877
	Dependent(Y)	0.631058	0.2333		0.147226	
Third Model	Independent (X)	0.794925		0.1626		0.129255
	Dependent(Y)	0.794925	0.3095		0.246029	
Fourth Model	Independent (X)	0.808603		0.1117		0.090321
	Dependent(Y)	0.808603	0.2252		0.182097	

CONCLUSION

It has been observed that the causal relationship between the courses of the Department of Statistics in the University of Benghazi depends on their subsequent order as shown by the four models division discussed in the analysis section. The four models were significant for the first removed root which approves that the independent variables (preceding variables) significantly affect the dependent variables (succeeding variables). The relationships among the courses of the Department of Statistics, (from a cause and affect point of view), was seen in the results of CCA and summarized eventually in the tables of mixed factor loadings. Those relationships reflect two types of directions; either the performance of students' grades in the two groups is increasing or decreasing on the average.

The interpretations made on the basis of Table 11 express many facts about the nature of the courses of the Department of Statistics. Some courses were taught for a long period of time by some lecturers who can be defined as "strict"; their results on the average reveal a low performance of the students such as $\mathbf{X_{56s7}}$ (Biostatistics), $\mathbf{X_{40s8}}$ (Applied Linear Models) and $\mathbf{X_{41st5}}$ (Experimental Design 2). On the other hand, some courses have a positive sign which means that the level of the student was high and that was a consequence of the lecturers of these courses who where more lenient such as $\mathbf{X_{43s8}}$ (Time Series Analysis), even though the nature of the course is not "too easy", but because of the lecturer who has taught the course for a long time the performance was increasing. Some courses, such as $\mathbf{X_{35s8}}$ (Graduation Project), give an idea about the positive performance of the students; this course usually ends in high grades (A's and B's), and that was because of the policy of the department to reward students for their efforts in their graduation research.

The courses which consist of two parts showed high and moderate positive loadings such as $\mathbf{X_{32st4}}$ (Distribution Theory) and $\mathbf{X_{31st5}}$ (Advanced Distribution Theory), $\mathbf{X_{42s4}}$ (Experimental Design 1) and $\mathbf{X_{41s5}}$ (Experimental Design 2), $\mathbf{X_{52s4}}$ (Sampling Techniques 1) and $\mathbf{X_{51s5}}$ (Sampling Techniques 2).

REFERENCES

1. Rencher, A.C (1998). *Multivariate Statistical Inference and Applications*, John Wiley and Sons, Inc. New York. Chapter 8, 312-333.
2. Abdelghffar F. Abdlghffar (2007). *Studying the Influences of Some Socio-Demographic Factors Simultaneous on Fertility and Child Mortality in Benghazi using Canonical Correlation Analysis*, Second Conference of Basic Sciences. Tripoli. 4-8-11.
3. Martin Bilodeau, David Brenner (1999). *Theory of Multivariate Statistics*, Springer, PP.174-190.

APPENDIX A

Table A1
The Canonical Weights of the First Removed Root for the Second Model

Variables	Canonical Weights for Independent Variables (X)	Canonical Weights for Independent Variables (Y)
X32s4	0.047960	
X36s3	0.390758	
X37s2	-0.069532	
X42s4	0.651964	
X47s4	-0.018640	
X49s1	-0.317382	
X50s4	-0.277181	
X52s4	0.149730	
X57s3	-0.103811	
X31s5		0.047985
X33s5		-0.180703
X41s5		0.555348
X45s5		-0.027510
X51s5		0.815487

Table A2
The Canonical Weights of the First Removed Root for the Third Model

Variables	Canonical Weights for Independent Variables (X)	Canonical Weights for Independent Variables (Y)
X31s5	0.096672	
X32s4	0.036894	
X33s5	0.118352	
X36s3	0.069556	
X37s2	-0.217089	
X41s5	-0.107101	
X42s4	0.290561	
X45s5	-0.148878	
X47s4	0.180410	
X49s1	-0.071442	
X50s4	-0.040478	
X51s5	-0.129644	
X52s4	-0.740447	
X57s3	-0.322981	
X34s6		0.365784
X38s6		-0.031562
X46s6		0.313136
X48s6		0.494661
X54s6		-0.382742
X55s6		0.096560

Table A3
The Canonical Weights of the First Removed Root for the Fourth Model

Variables	Canonical Weights for Independent Variables (X)	Canonical Weights for Independent Variables (Y)
X31s5	0.033046	
X32s4	-0.021291	
X33s5	0.300022	
X34s6	0.143650	
X36s3	-0.498840	
X37s2	-0.158428	
X38s6	-0.284733	
X41s5	-0.466716	
X42s4	0.142101	
X45s5	0.131508	
X46s6	-0.037952	
X47s4	-0.385952	
X48s6	-0.551177	
X49s1	-0.138461	
X50s4	0.559556	
X51s5	-0.324039	
X52s4	0.231969	
X54s6	-0.003379	
X55s6	0.235425	
X57s3	-0.435831	
X35s8		0.400514
X39s7		-0.233473
X40s8		-0.168706
X43s8		0.455423
X44s7		-0.192301
X56s7		-0.670769

Table A4
The Canonical Factor Loadings of Groups and the Percentage of Explained Variance for Each Group with the Corresponding Canonical Variable for the Second Model

Variables	Canonical Factor Loadings for Independent Variables (X)	Canonical Factor Loadings for Dependent Variables (Y)
X32s4	0.518126	
X36s3	0.615581	
X37s2	0.180920	
X42s4	0.825576	
X47s4	-0.212324	
X49s1	0.025502	
X50s4	-0.488505	
X52s4	0.572198	
X57s3	0.077076	
X31s5		0.255777
X33s5		0.172038
X41s5		0.592094
X45s5		0.046636
X51s5		0.847689
Variance Explained	21.99%	23.33%
Redundancy	13.88%	14.7%

Table A5
The Canonical factor Loadings of Groups and the Percentage of Explained Variance for Each Group with the Corresponding Canonical Variable for the Third Model

Variables	Canonical factor loadings for Independent Variables (X)	Canonical Factor Loadings for Dependent Variables (Y)
X31s5	0.212076	
X32s4	-0.394219	
X33s5	0.209287	
X36s3	-0.584037	
X37s2	-0.417237	
X41s5	-0.044410	
X42s4	0.010883	
X45s5	-0.182122	
X47s4	0.523802	
X49s1	-0.257706	
X50s4	0.403034	
X51s5	-0.308808	
X52s4	-0.782453	
X57s3	-0.521055	
X34s6		0.680567
X38s6		0.416523
X46s6		0.654956
X48s6		0.731856
X54s6		-0.464612
X55s6		0.199550
Variance Explained	16.26%	30.95%
Redundancy	12.93%	24.60%

Table A6
The Canonical factor Loadings of Groups and the Percentage of Explained Variance for each Group with the Corresponding Canonical Variable for the Fourth model.

Variables	Canonical Factor Loadings for Independent Variables (X)	Canonical Factor Loadings for Dependent Variables (Y)
X31s5	-0.043507	
X32s4	-0.456382	
X33s5	0.058772	
X34s6	0.044917	
X36s3	-0.503523	
X37s2	-0.286075	
X38s6	0.178660	
X41s5	-0.658066	
X42s4	-0.418197	
X45s5	0.203848	
X46s6	0.230101	
X47s4	0.311453	
X48s6	-0.068490	
X49s1	-0.225565	
X50s4	0.648106	
X51s5	-0.269908	
X52s4	-0.220182	
X54s6	-0.351476	
X55s6	-0.151206	
X57s3	-0.327706	
X35s8		0.267844
X39s7		-0.253507
X40s8		-0.589463
X43s8		0.567423
X44s7		0.085795
X56s7		-0.733745
Variance Explained	11.17%	22.52%
Redundancy	9.03%	18.21%