SHEWHART CONTROL CHARTS FOR RAYLEIGH DISTRIBUTION IN THE PRESENCE OF TYPE I CENSORED DATA

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ABSTRACT

This article explains shewhart control charts for monitoring the mean level of the Rayleigh lifetimes under the type I censored data. The control chart for the type I censored data is developed based on the conditional expected values (CEV). The results of CEV based control charts are compared with simple/traditional shewhart structures. The CEV control chart outperforms traditional control chart when the data is censored. The proposed method is illustrated by an example.

KEY WORDS

Type I censored, CEV, Shewhart control charts.

1. INTRODUCTION

Grant and Leavenworth (1979) describe the “Statistical Process Control (SPC)” as a “useful and important tool used commonly in engineering field to monitor the overall process.” SPC can be applied to all kinds of engineering operations. The significant application of the SPC analysis of the process will make the process more consistent and reliable.

Statistical Process Control is usually referred as “SPC”. It is a method which includes:

- Monitoring.
- Controlling.
- Refining.

The real life application clearly states that all processes inherent some variation. But sometimes the process shows a great level of discrepancy and results in the occurrence of offensive/unpredictable results. One of the uses of the SPC is to reduce variation to achieve the best objective value.

The Control charts are the most important tool of SPC tool kit. It is commonly used to differentiate between the “assignable and un-assignable causes”. The purpose of the effective process monitoring system is to detect the presence of an “assignable cause” as
rapidly as possible without stopping checking too often or too late. The control charts are of different types. Some are “memory control charts” and other is “memory-less control charts”. Shewhart are memory-less control charts and are being used to detect a large size shift whereas the memory type charts are used for dealing with small size shifts.

The industrial tools are emerging rapidly therefore, it is essential to design the products with high consistencies. By using the highly censored data collected from life time distribution the time and expenses can be minimized. An important issue in life-testing applications for industrial engineering is how we can develop the control chart for monitoring the mean life time of products when the data is censored under Type-I.

Lu and Tsai (2008) has done work on Type I censored data using Gamma distribution and proposed EWMA conditional expected values (CEV) control chart for monitoring mean level of the gamma life times under type I censored test.

Steiner and Mackay (2000, 2001a) developed a one-side charting procedure based on the CEVs which allows for rapid detection of deterioration in the process quality with highly censored data under normality.

Tsai and Lin (2009) proposed a EWMA control chart to identify mean shifts for the Gompertz distributed lifetimes with the decrease and increase in type I censoring. Zhang and Chen (2004) shows the practical implementation of censored data analysis in which we monitor a painting process regarding the rust resistant capabilities, scratch panels from a type of metal electrical box painted using this painting process are put in a salt spray chamber with the temperature maintained at 30 Celsius. It was concluded that when the data is censored the simple/traditional control charting such as $\bar{X}$ and $R$ charts shows as large false alarm rates or low power.

Simple/traditional Control charting methodology couldn’t provide an effective analysis in the presence of censored data therefore to solve this issue statistician/ researchers have developed different methods for different life time distributions. In this paper a methods is discussed which provides an effective results in the presence of censored data for Rayleigh distribution. This paper is organized as following sections: The introduction is given in Section 1, in section 2 proposed methodology is presented and in section 3 Numerical computations for conditional expected values (CEVs) and charting tools are presented. Section 4 covers some concluding remarks about proposed methods.

2. THE PROPOSED SHEWHART CEV CONTROL CHARTS

The CEV weighted control charts are used for detecting mean level shifts in the process. Using CEVs weighted control charts each censored observation is replaced with its condition expected values. Now the test statistic of control chart is calculated. For our case the subgroup averages and standard deviations are calculated and plotted in a manner similar to the traditional $\bar{X}$ and $S$ charts. With right censored data the goal of CEV weighted control charts is to detect decrease in process mean and/or increase in process standard deviation. The proposed control charts are both one sided. We have considered decrease in mean level shifts and increase in standard deviation shifts using concept given by Steiner and Mackay (2000). The CEV $\bar{X}$ charts have only lower sided
control limits whereas the CEV $S$ charts have only upper sided control limits. In this paper we have calculated CEV based control charts and also performed their comparisons with previously used simple shewhart control charts.

The procedural details for constructing CEV based $\overline{X}$ and $S$ charts are given as:

Let the lifetime of items $T_{i1}, T_{i2}, \ldots, T_{in}$ (Where “$i$” show subgroup number and “$n$” show the sample size) follows Raleigh distribution; all items are put on type I Censoring test. The lifetimes of items are exactly known only if they are less than or equal to the censoring time $C$. The practitioners predetermined the censoring time. The mean lifetime of $C$ is presented as $\mu_0 = 1.25\sigma$.

The censoring rate is defined as $P_c = 1 - F(t; \sigma)$, where $F(t; \sigma)$ is the cumulative density function of Raleigh distribution and is given as $P(T \leq t) = 1 - \exp\left(-t^2/(2\sigma^2)\right)$. If the process is under a statistical control state, the mean lifetime is assumed to be $\mu_0 = 1.25\sigma$. If the process parameter $\sigma$ is unknown it can be replaced by its MLE based on $m$ pre-samples each of size $n$. The CEV are derived as follows:

$$E(T|T > c) = \frac{1}{F(c, \alpha)} \int_c^{\infty} t \left(\frac{2t}{\alpha} e^{-\frac{t^2}{\alpha}}\right) dt = \frac{\alpha \Gamma(z.c, 3/2)}{\exp(-z.c)}$$

So CEV for Raleigh distribution: $E(T|T > c) = \frac{\alpha \Gamma(z.c, 3/2)}{\exp(-z.c)}$ where $\alpha > 0$.

And $z_c = \left(c / \alpha\right)^2$, $c$ is the censoring time and $\alpha = \left(\sqrt{2}\right)(\sigma)$.

Now we transfer the type I censored data set to:

$W_{ij} = \begin{cases} T_{ij}, & \text{if } T_{ij} \leq C \text{ (uncensored).} \\ CEV(T_{ij}), & \text{if } T_{ij} > C \text{ (censored).} \end{cases}$

$j = 1, 2, 3, \ldots, n$, $i = -1, 2, 3, \ldots, m$.

Now using the transformed distribution (i.e. the distribution of $W_{ij}$. ) is used to create One-sided CEV based $\overline{X}$, $S$ charts respectively, by plotting the subgroup averages and standard deviations.

Control Limits of control chart:

$UCL: \overline{X} + K \sigma_{\overline{X}}$

$CL: \overline{X}$

$LCL: \overline{X} - K \sigma_{\overline{X}}$

whereas $K$ is 99.73 quartile points of the Rayleigh distribution and its average values is calculated as 2.6.
3. NUMERICAL STUDY AND EXAMPLE

The following results have been computed using Rayleigh distribution with scale parameter $\sigma = 0.5$:

Table 1

<table>
<thead>
<tr>
<th>$P_c$</th>
<th>30% Decrease Mean (CEV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>40</td>
</tr>
<tr>
<td>0.3</td>
<td>32</td>
</tr>
<tr>
<td>0.5</td>
<td>30</td>
</tr>
<tr>
<td>0.6</td>
<td>22</td>
</tr>
</tbody>
</table>

The Table 1 indicates that the out-of-control ARLs (ARL$_i$) for the proposed CEV $\bar{X}$ control charts with different censoring rates. It is observed that the performance of the proposed CEV $\bar{X}$ control chart is preferable for high censoring rates.

Table 2

<table>
<thead>
<tr>
<th>$P_c$</th>
<th>30% Decrease Mean (CEV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>55</td>
</tr>
<tr>
<td>0.3</td>
<td>45</td>
</tr>
<tr>
<td>0.5</td>
<td>43</td>
</tr>
<tr>
<td>0.6</td>
<td>40</td>
</tr>
</tbody>
</table>

The Table 2 shows that the out-of-control ARLs for the proposed shewhart S control charts with different censoring rates. It is observed that the performance of the proposed CEV S control chart is preferable for high censoring rates.

Table 3

<table>
<thead>
<tr>
<th>$P_c$</th>
<th>CEV $\bar{X}$ chart (Mean (CEV))</th>
<th>Traditional $\bar{X}$ chart (ignoring censoring)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>40</td>
<td>47</td>
</tr>
<tr>
<td>0.3</td>
<td>34</td>
<td>58</td>
</tr>
<tr>
<td>0.5</td>
<td>30</td>
<td>67</td>
</tr>
<tr>
<td>0.6</td>
<td>22</td>
<td>72</td>
</tr>
</tbody>
</table>

Table 4

<table>
<thead>
<tr>
<th>$P_c$</th>
<th>CEV $S$ chart (Mean(CEV))</th>
<th>Traditional S chart (ignoring censoring)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>55</td>
<td>72</td>
</tr>
<tr>
<td>0.3</td>
<td>45</td>
<td>74</td>
</tr>
<tr>
<td>0.5</td>
<td>43</td>
<td>77</td>
</tr>
<tr>
<td>0.6</td>
<td>40</td>
<td>84</td>
</tr>
</tbody>
</table>
The Table 3 given below shows that $CEV \bar{X}$ control charts performs well as compared to traditional control charts. Similarly the Table 4 given below shows that $CEV S$ control charts performs well as compared to traditional control charts in presence of censoring data. It is also observed that for high censoring rates increases then results get more preferable for proposed Censoring control charts.

**Graphical Displays**

The figure given below shows the performance of proposed censoring control chart to traditional Shewhart control charts.

![Figure 1: Plot of CEV $\bar{X}$ Chart vs. Traditional $\bar{X}$ Chart (Ignoring Censoring)](image)

Figure 1: Plot of CEV $\bar{X}$ Chart vs. Traditional $\bar{X}$ Chart (Ignoring Censoring)

![Figure 2: Plot of CEV S Chart vs. Traditional S Chart](image)

Figure 2: Plot of CEV S Chart vs. Traditional S Chart

The Figure 1 & 2 shows that censored control charts performs better than traditional control charts.
The Figure 3 shows that after 30% shift is introduced in mean the assignable cause is detected at 132 subgroup number, so the $ARL_1 = 32$ for 30% censoring rate.

The Figure 4 shows that for S chart, when 30% shift is introduce in mean the assignable cause is detected at 145 subgroup number. So the $ARL_1 = 45$ for 30% censoring rate.
Figure 5: $ARL_1$ for CEV $\bar{X}$ Chart for 30% Mean Decrease and $P_c = 0.6$

The Figure 5 shows for 30% shift in mean the assignable cause is detected at 122 subgroup number, so the $ARL_1 = 22$ for 60% censoring rate.

Figure 6: $ARL_1$ for CEV $S$ Chart for 30% Mean Increase and $P_c = 0.6$

The Figure 6 shows that the assignable cause is detected at 140 subgroup number for $S$ chart using 30% shift in mean, so the $ARL_1 = 40$ for 60% censoring rate.

4. CONCLUSION

In manufacturing industries, we may observe the censored data of type-I. In the presence of the censored data the existing/traditional control charts like $\bar{X}$ and $S$ control charts, has large false alarm rate resulting in low power. In this paper, CEV Control charting is used to monitor the process average and standard deviation, when the
observations are censored at fixed Levels. The comparison shows that the CEV control charts outperform traditional control charts in the presence of censored data. It is also observed that with the increase in censoring rates the results get more preferable for proposed CEV control charts.

REFERENCES